

## ISOLATED MAGNETAR SPIN-DOWN, SOFT X-RAY EMISSION AND RX J1856.5-3754

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## ABSTRACT

When an isolated magnetar with magnetic dipole field  $B \sim 10^{15}$  G moves at high velocity ( $v > 10^7$  cm s<sup>-1</sup>) through the ISM, its transition into propeller effect driven spin-down may occur in less than  $10^6$  years. We propose that the nearby neutron star RX J1856.5-3754 is such a magnetar, and has spun down by the propeller effect to a period greater than  $10^4$  sec within  $\sim 5 \times 10^5$  years. This magnetar scenario is consistent with observed thermal X-ray emission properties and the absence of detectable spin-modulations of them. Detection of other rapidly moving long period ( $> 100$  sec) magnetars with known ages would strongly constrain the very great variety of predicted propeller-effect torque magnitudes.

*Subject headings:* stars: neutron – X-rays: stars – stars: RX J1856.5-3754

## 1. INTRODUCTION

Isolated magnetars (spinning-down neutron stars with dipole fields  $10^{14}$ – $10^{15}$  G) differ from canonical ( $10^{11}$ – $10^{13}$  G) pulsars in several ways:

- (1) they spin-down very much more rapidly;
- (2) when relatively young (age  $\lesssim 10^6$  years), their spin-down torque mechanism may change over from the relativistic particle plus magnetic field wind of canonical pulsars to a “propeller” interaction (Davidson & Ostriker 1973; Illarionov & Sunyaev 1975) on a surrounding interstellar medium if they are rapidly moving through it;
- (3) their strongly magnetized surface can have very different emissivities for extraordinary and ordinary mode thermal X-rays.

We consider below the extraordinarily varied spin-down descriptions for magnetars when various proposed propeller interaction models are used (Illarionov & Sunyaev 1975; Davies et al. 1979). Especially significant may be a possible almost complete quenching of the spin of some magnetars within  $10^6$  years, not possible for isolated canonical pulsars or for magnetars embedded in their own accretion disks. We apply such considerations to understanding the observed X-ray emission from the rapidly moving isolated  $10^6$  year old neutron star RX J1856.5-3754 (hereafter RX J1856). It has featureless nearly blackbody thermal X-ray emission (with a paradoxically small blackbody radius) which shows no indication of spin-period modulation ( $< 1\%$ ) in the range  $10 \text{ ms} < P < 10^4 \text{ sec}$  (Ransom et al. 2002; Drake et al. 2002; Burwitz et al. 2003).

Four possible explanations of the lack of detected spin-modulation of the surface X-ray emission of an isolated  $10^6$  year old NS are

- (1) Our line of sight to this pulsar is almost exactly along the pulsar spin-axis (Braje & Romani 2002);
- (2) the magnetic field of this neutron star, unlike that of all known pulsars, is almost exactly axially symmetric with respect to the neutron stars spin-axis (Braje & Romani 2002);
- (3) this particular neutron star was born less than  $10^6$  years ago as a weak field millisecond pulsar ( $P < 10 \text{ ms}$ )

(4) this neutron star’s spin has been almost quenched to  $P > 10^4 \text{ sec}$ . Of these (4) may be the most plausible. Whatever the details of the mechanism for achieving it are, the needed effective spin-down torque at very long periods seems to require both a transition out of the canonical pulsar spin-down mechanism, where energy and angular momentum are lost from an almost corotating magnetosphere at its very distant ( $\lesssim 10^{14} \text{ cm}$ ) light cylinder radius, and a huge surface dipole magnetic field. These two requirements encourage exploring the possibility that RX J1856 is a magnetar, rapidly moving through the ISM or a molecular cloud, is now spinning-down by propeller ejection of that incident matter.

The propeller effect may be an efficient spin-down mechanism when the spin-period becomes larger than seconds, but proposed spin-down torques from it are highly model-dependent. We find that RX J1856 should spin-down to a period  $> 10^6 \text{ sec}$  for some proposed propeller-effect models. The magnetar scenario is also consistent with observed spectral properties for both a single temperature and two-component blackbody model.

## 2. SPIN-DOWN EVOLUTION

When an isolated neutron star (NS) is spun-down by the torque of its relativistic wind emission,

$$I\dot{\Omega} \sim -\frac{\mu^2\Omega^3}{c^3} \equiv -K\Omega^n, \quad (1)$$

where  $I$  is the NS moment of inertia,  $\mu$  is the NS magnetic dipole moment and  $\Omega$  is the NS spin-velocity. The “braking index”  $n$  is 3 if  $\mu$  and  $\vec{\mu} \cdot \vec{\Omega}/\Omega$  are constant. When the NS is born with a spin-period much shorter than a presently observed one, the age of the star is the “spin-down” age  $\tau_{sd} \equiv \frac{P}{(n-1)\dot{P}}$ , where  $P$  and  $\dot{P}$  are the present spin period and its time derivative. The canonical spin-down age,  $\tau_{csd} \equiv P/2\dot{P}$ , differs from the true age for  $n \neq 3$ . If  $n < 1$   $\tau_{sd}$  clearly must have a very different meaning. When  $n > 1$ , the canonical spin-down age indicates the time since the NS entered the spin-down phase described by equation (1):

$$\tau_{sd} = \frac{2}{n-1}\tau_{csd}, \quad (2)$$

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as long as the original spin-period  $\ll P$  but when  $n < 1$ , the canonical spin-down age indicates *the approximate time remaining before the NS spin will be completely quenched* ( $\tau_f$ ):

$$\tau_f = \frac{2}{1-n} \tau_{csd}. \quad (3)$$

To quench a NS's spin, that spin-down must then be in a phase described by  $n < 1$  in equation (1).

When the NS light cylinder radius ( $c/\Omega$ ) becomes larger than the radius of its corotating magnetosphere ( $R_m$ ), the spin-down mechanism is changed to one in which the spin-down torque is from direct spin-up of the surrounding medium ("propeller effect"). In this case, the braking index is no longer  $\sim 3$ . In proposed models, it is generally in the range  $-1 \leq n \leq 2$ .

Propeller effect spin-down torque depends upon three parameters, the NS magnetic dipole moment ( $\mu$ ), the density of incoming gas at the magnetosphere boundary with the surrounding medium ( $\rho_m$ ) and velocity of the incoming gas at the magnetosphere boundary in the NS rest frame ( $v_m$ ). Assuming the NS spin-down torque has fixed power-law dependences on these parameters, dimensional analysis gives

$$I\dot{\Omega} = -\kappa \mu^{(3+n)/3} \rho_m^{(3-n)/6} v_m^{(3-4n)/3} \Omega^n \quad (4)$$

with  $\kappa$  and  $n$  dimensionless constants. In the following discussion, we set NS mass and radius to  $1.4M_\odot$  and 10 km respectively.

### 2.1. Models for propeller caused spin-down

Two parameters can classify the many proposed propeller spin-down models. In these models, spin-down torque is

$$I\dot{\Omega} = -\dot{M} R_m v_m \mathcal{M}^\gamma, \quad (5)$$

where  $\dot{M} = \rho_m (\pi R_m^2) v_m$  is the incoming (and largely ejected) mass rate.  $\mathcal{M}$  is the Mach number defined as the ratio of incoming medium velocity to NS spin-velocity at the magnetosphere boundary:  $\mathcal{M} \equiv R_m \Omega / v_m$ . Proposed propeller models have  $\gamma$  between  $-1$  and  $2$ .  $\mathcal{M} > 1$  and  $\mathcal{M} < 1$  are the supersonic and subsonic regimes, which have been treated differently in proposed models.

The ram pressure balance at the (non-spherical) magnetosphere boundary from either  $v_m$  or  $\Omega R_m$  is assumed to be

$$\frac{B_m^2}{8\pi} = \rho_m v_m^2 \mathcal{M}^\delta, \quad (6)$$

where  $B_m$  is local magnetic field strength at the magnetosphere boundary. Since  $B_m = \mu / R_m^3$  for a dipole magnetic field with dipole moment  $\mu$ , the braking index  $n$  is then

$$n = \frac{3(2\gamma - \delta)}{\delta + 6}. \quad (7)$$

The spin-down torque is

$$I\dot{\Omega} = -\frac{\mu^2}{R_m^3} \mathcal{M}^{\gamma-\delta}. \quad (8)$$

A remarkably large number of different propeller spin-down models have been proposed and applied. The braking indices suggested by these propeller models are in the range of  $-1 \leq n \leq 2$ . Some of the models were intended only for NSs with spin-down from surrounding accretion

disks. Table 1 categorizes most of the proposed propeller effect models by their  $\gamma$  and  $\delta$ .

We note that at the transition into propeller spin-down of an isolated pulsar,  $\Omega R_m \sim c$  so that  $\mathcal{M} \sim 10^3$  in equations (5), (6) and (8). In the models of Table 1 (by various authors, and, sometimes, the same authors at different time) the  $\gamma$ - $\delta$  exponent in equation (8) varies from  $-1$  to  $2$  corresponding to differences of  $10^9$  in the magnitude of the propeller driven spin-down torques which take over from that of equation (1).

In most models  $\gamma \neq \delta$  so that in the transition from pulsar to propeller phase there is a huge discontinuity in spin-down torque. We favor a plausible propeller spin-down model with  $\gamma = \delta$  which gives no such discontinuity.

A particularly simple model completely ignores all magnetic fields within both the incoming and ejected ionized beams and also possible collisionless collection stream-stream interactions. Because both flows are so very dilute ( $n_m \lesssim 10 \text{ cm}^{-3}$ ) single particle interaction mean free paths  $\gg R_m (\sim 10^{12} \text{ cm})$ . Incidence and reflection of the ISM on the stellar magnetosphere could then be described as a sum of single particle trajectories. A "rough" interface model (RIM) is one in which the interface between the corotating magnetosphere of the NS and the ISM flow onto it is approximated as a sawtooth-like structure in the corotating frame, with angles between the sawtooth surfaces and the surface of constant radius from the star assumed to be of order a radian. In "smooth" interface models that interface is assumed to be much more spherical (typically with angular deviations of order the inverse Mach number of the very supersonic propeller motion). In a "rough" interface model the deflected incident flow pushes strongly with a comparable force in the tangential and radial directions. This gives  $\gamma = \delta = 1$  and  $n = 3/7$ . (In a smooth interface limit,  $\gamma = -1, \delta = 0$  gives the  $n = -1$  of Illarionov & Sunyaev (1975)). In our discussions below we give particular emphasis to it.

### 2.2. Transitions between different spin-down stages of magnetars

We discuss three transitions in the spin evolution of magnetars as  $\Omega$  decreases. When  $\Omega = c/R_m$  there is a transition from canonical pulsar spin-down to propeller spin-down. When  $\Omega = v_m/R_m$ , there is the transition from supersonic ( $\mathcal{M} > 1$ ) to subsonic propeller spin-down ( $\mathcal{M} < 1$ ). When  $\Omega = \Omega_K(R_m)$  (the Keplerian angular velocity at the magnetosphere boundary), there is a transition from propeller spin-down to strong accretion. Table 2 summarizes spin-periods and transition times for different propeller spin-down models.

## 3. THE EXCEPTIONAL ISOLATED NEUTRON STAR RX J1856

The isolated neutron star RX J1856 was serendipitously discovered by *ROSAT* as an X-ray source with thermal emission (Walter et al. 1996). Detection of proper motion provided a distance  $\sim 120 \text{ pc}$  and velocity  $\sim 200 \text{ km s}^{-1}$  and supports an association of RX J1856 with the Upper Scorpius birthplace indicating a NS age  $\sim 5 \times 10^5 \text{ yrs}$  (Walter & Lattimer 2002; Kaplan et al. 2002). Deep *Chandra* and *XMM-Newton* observations with total exposure time  $\sim 500 \text{ ks}$  were performed to search for spectral feature and

pulsation. However, neither was found in the X-ray data which had high resolution and sensitivity (Ransom et al. 2002; Drake et al. 2002).

This lack of observed pulsation is perplexing since modulation of an anisotropic temperature distribution suggested by the two-component blackbody model fit discussed below seems expected, and pulsation modulation of X-ray emission has been detected in other isolated NSs with thermal emission (Haberl et al. 1997; Zavlin et al. 2000). We propose that RX J1856 has been spun-down to a spin period longer than  $10^4$  sec by the propeller effect. The fraction of magnetars among isolated NSs is  $\sim 10^{-1}$  (Kouveliotou et al. 1998), and is much larger among NSs from which thermal emission has been detected. We find that the only plausible way to achieve the needed condition for early transition into the propeller phase and rapid enough spin-down thereafter is for this star to be a magnetar.

We first assume  $B \geq 10^{14}$  G, sufficiently large that transition into the propeller phase occurs in much less than  $5 \times 10^5$  years. Then, we searched parameter space in the  $\gamma$ - $\delta$  plane for the region in which RX J1856 could, thereafter, have been spun-down to a period longer than  $10^4$  sec. Input parameters are magnetic dipole moment  $\mu$ , incoming gas density  $n_m$ , and incoming gas velocity  $v_m$ . We fixed  $v_m$  from the observed stellar proper motion at 200  $\text{km}^{-1}$ .

ISM particles (mainly hydrogen) at the magnetosphere radius will be charged and be reflected by the stellar magnetosphere. [Thermal photons from the NS surface ionize all hydrogen in the vicinity of the star. The ionization time of hydrogen at  $R_m \sim 10^{12}$  cm ( $\sim 10^2$  s) is much shorter than  $R_m/v_m$  ( $\sim 10^5$  s). Therefore, hydrogen is fully-ionized at the magnetosphere boundary before the star reaches that region].

Figure 1 shows the  $\gamma$ - $\delta$  parameter space for which RX J1856 can spin-down to  $P > 10^4$  sec within  $5 \times 10^5$  years. Our rough interface model ( $\gamma = \delta = 1, n = 3/7$ ) can achieve the required spin-down for  $B \sim 5 \times 10^{15}$  G ( $\mu_{33} = 5$ ) and  $n_m = 1 \text{ cm}^{-3}$  and  $B \sim 5 \times 10^{14}$  G ( $\mu_{33} = 0.5$ ) and  $n_m = 10^5 \text{ cm}^{-3}$  (estimated density of the nearby molecular cloud R CrA through which RX J1856 may have passed (Giannini et al. 1998)).

### 3.1. Thermal emission from RX J1856

The surface temperature of RX J1856 is roughly consistent with predictions of standard cooling curves at its age  $\sim 5 \times 10^5$  years (Tsuruta et al. 2002). RX J1856 may radiate because of accretion at an incoming mass rate  $\dot{M} = \pi R_m^2 v_0 \rho_m = 5 \times 10^7 R_{m,12}^2 v_{m,7} n_m \text{ g s}^{-1}$ . However, the X-ray luminosity from such an accretion rate is below the detection limit of *Chandra* and *XMM-Newton* observations (Rutledge 2001). A third heat source may be the continuous dissipation of the large magnetic field energy in the stellar crust (Heyl & Kulkarni 1998).

Given featureless X-ray spectra, several approaches using spectral energy distribution (SED) in the optical and X-ray band have been undertaken to reveal the surface composition (Pons et al. 2002). Light element atmospheres (H and He) were ruled out since they predict about two orders of magnitudes larger optical flux compared to the observed values (Pons et al. 2002). On the other hand,

a blackbody model underpredicts optical flux by a factor of  $\sim 7$  (Walter & Lattimer 2002). Non-magnetized heavy element atmosphere models (e.g. Si-ash or Iron) predict correct SED over the optical and X-ray band with a single temperature ( $kT^\infty = 40$  eV) (Walter & Lattimer 2002). However, they have absorption features which deviate from the featureless X-ray data (Burwitz et al. 2003). A similar situation exists for magnetized heavy element atmospheres (Rajagopal et al. 1997).

A two-component blackbody model was proposed to account for the multi-wavelength SED and the featureless X-ray spectra (Pons et al. 2002). Hot and cold blackbody components for X-ray and optical spectra respectively are consistent with both SED and featureless X-ray data (Pons et al. 2002; Braje & Romani 2002). However, it is hard to obtain blackbody-like spectra when significant atmosphere is present on the surface.

At sufficiently high magnetic field strength and low temperature, a NS surface becomes very dense liquid or solid with almost no atmosphere above it (Ruderman 1971; Lai & Salpeter 1996).

That RX J1856 is a magnetar is consistent with the two temperature model (anisotropy of surface temperature distribution caused by strong magnetic field) and the model for a condensed iron surface because they both require strong magnetic field strength on the surface. The emissivity of such a condensed matter surface in  $10^{15}$  G is about 1/2 that of a blackbody because the strongly magnetized very dense ( $\rho \sim 560 Z^{-3/5} B_{12}^{6/5} \text{ g cm}^{-3}$ , with  $B = 10^{12} B_{12}$  G) has essentially no emissivity for O-mode X-rays but is near blackbody for E-mode ones. Then the average emissivity for the radiating surface is  $\sim 30$ –50% depending on photon energy and magnetic field geometry (Zane et al. 2003). The actual NS area should then be more than twice that of the apparent blackbody area so that the inferred NS radius becomes  $\gtrsim \sqrt{2} R_{BB}$  (Zane et al. 2003).

## 4. DISCUSSION

Will we find other isolated NSs in the propeller phase? There are several NSs with discrepant supernova remnant and “canonical” spin-down ages ( $\tau_{csd}$ ). In some cases, the discrepancy may be due to the fact that NSs are in a propeller phase. (Alternatively, they could have been born with a long spin period close to present value.) However, some of them have spin periods shorter than  $< 1$  sec, so it is difficult for them to enter a propeller phase unless the ambient gas density is very large ( $n_m \gg 1 \text{ cm}^{-3}$ ).

Detection of long pulsation periods ( $> 10$  sec) from isolated NSs may be another indication of NSs in a propeller phase. Potential candidates for such isolated NSs would have ages between  $10^3$  years (enough time for spinning down to enter the propeller phase) and  $10^6$  years (still detectable thermal emission). In a magnetar, magnetic field decay processes can keep the magnetar more X-ray luminous than would be the case for canonical pulsars (Heyl & Kulkarni 1998). Then older magnetars might be still observable by their X-ray emission after canonical NSs have faded.

Detection of spin-period variation in X-ray flux over several X-ray observations would explore periods  $> 10^4$  sec for the case of RX J1856. In most cases in which RX J1856 has spun-down to  $P > 10^4$  sec (§3), it should at present

have a period of at least  $10^6$  sec. The chance of seeing a different surface aspect of the star may be small and gravitational light-bending effects will smear out some spectral modulation.

If an isolated NS in a propeller phase is confirmed it will select among propeller effect models. If a NS is in the propeller phase, one should be cautious in deriving NS ages and magnetic field strengths from  $P$  and  $\dot{P}$  measurements since the conventional dipole radiation wind formulae of equation (1) and (2) are no longer valid. In such cases, independent determination of ages and magnetic field strengths (e.g. supernova remnant ages and spectroscopic measurements of B-field) are needed.

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TABLE 1  
PROPELLER EFFECT MODEL PARAMETERS.

$\gamma$	$\delta$	$n$	$n_\mu$	$n_\rho$	$n_v$	Models
-1	0	-1	2/3	2/3	7/3	Illarionov & Sunyaev (1975)
0	0	0	1	1/2	1	Davidson & Ostriker (1973)
1	0	1	4/3	1/3	-1/3	Menou et al. (1999)
2	0	2	5/3	1/6	-5/3	Davies et al. (1979)
1	1	3/7	8/7	3/7	3/7	A rough interface model (RIM)
2	2	3/4	5/4	3/8	0	Romanova et al. (2002) <sup>a</sup>

<sup>a</sup>Their analytic model gives  $n = 3/5$  which differs from the  $n = 3/4$  because they used the outflow density which depends on  $\Omega R_m$  (see more details in section 2 of Romanova et al. (2002)). In their numerical model,  $n = 1.3$  and  $n_\mu = 0.8$ .

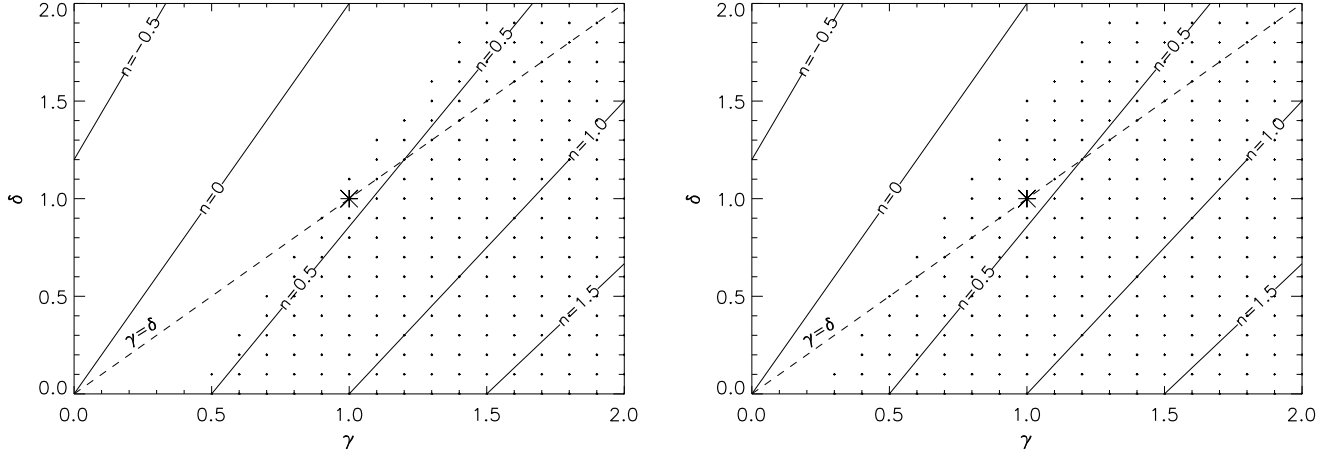


FIG. 1.—  $\gamma$ - $\delta$  diagram for  $\mu_{33} = 5$  and  $n_m = 1 \text{ cm}^{-3}$  (left) and  $\mu_{33} = 0.5$  and  $n_m = 10^5 \text{ cm}^{-3}$  (right). The dotted region shows the  $\gamma$ - $\delta$  parameter space for which RX J1856 can spin-down to  $P > 10^4$  sec within  $5 \times 10^5$  years. The asterisk refers to our favorite model ( $\gamma = \delta = 1$ ). The ISM density in the vicinity of RX J1856 has been estimated to be  $\sim 1 \text{ cm}^{-3}$  from studies of the nearby H $\alpha$  nebula (van Kerkwijk & Kulkarni 2001). RX J1856 may have passed through the nearby molecular cloud R CrA with density  $> 10^5 \text{ cm}^{-3}$  (Giannini et al. 1998).

TABLE 2  
NS PERIODS AND TRANSITION AGES FOR DIFFERENT PULSAR SPIN-DOWN MECHANISMS.

Transition	Period [sec]	Transition time [yrs] <sup>a</sup>
Pulsar wind $\rightarrow$ Propeller	$37 \mu_{33}^{1/3} n_m^{-1/6} \beta_m^{(\delta-2)/6}$	$2.2 \times 10^4 \mu_{33}^{-4/3} n_m^{-1/3} \beta_m^{(\delta-2)/3}$
Supersonic $\rightarrow$ Subsonic	$1.6 \times 10^6 \mu_{33}^{1/3} n_m^{-1/6} v_{m,7}^{-4/3}$	$[10^5-10^7]^b$
Propeller $\rightarrow$ Accretion	$\sim [10^6]^b$	$[10^5-10^7]^b$

Note. —  $\beta_m \equiv v_m/c$  and  $\mu = 10^{33} \mu_{33} \text{ G cm}^3$ .  $\mu_{33} = 1$  corresponds to  $B = 10^{15} \text{ G}$  and  $R = 10 \text{ km}$ .

<sup>a</sup>Age since birth.

<sup>b</sup>These do not usually have simple power-law forms and we show approximate ranges. We assumed  $\gamma = \delta = 1$ . The parameter ranges we considered here are:  $B \sim 10^{14}-10^{15} \text{ G}$ ,  $n_m \sim 1-10 \text{ cm}^{-3}$  and  $v_m \sim 10-100 \text{ km s}^{-1}$ .